Calc AB Ch 5 Test Practice

Name: Block: Seat:

Chapter 5 Check List:

1 ...... Definition of Natural Log Function (p. 314)

2 ...... Natural Log Properties (pp. 315-316)

3 ...... Derivative of the Natural Logarithmic Function (p. 318)

4 ...... $\dfrac{d}{dx}[\ln|u|] = \dfrac{u'}{u}$ (p. 320)

5 ...... $\displaystyle\int \frac{u'}{u}\,dx = \ln|u| + C$ (p.324)

6 ...... Integrals of the 6 Basic Trig Functions (p. 329) including

$$\int \tan u\,du = -\ln|\cos u| + C$$

$$\int \cot u\,du = \ln|\sin u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc u\,du = -\ln|\csc u + \cot u| + C$$

7 ...... Derivative of an Inverse Function (p. 337)
If $f(g(x)) = x$, $g'(x) = \dfrac{1}{f'(g(x))}$

8 ...... Def. of Natural Exponential Function (p. 342)

9 ...... Properties if e^x (p 343)

10 ...... $\dfrac{d}{dx}[e^u] = e^u \cdot u'$ (p. 344)

11 ...... $\displaystyle\int e^u\,du = e^u + C$ (p.346)

12 ...... Def. of Exp and Log with Base a:
$a^x = e^{(\ln a)x}$ (p. 352)
$\log_a x = \dfrac{1}{\ln a}\ln x$ (p. 353)

13 ...... Properties if Inverse Functions (p 353)

14 ...... $\dfrac{d}{dx}[a^u] = (\ln a)a^u \cdot u'$ (p. 354)

15 ...... $\dfrac{d}{dx}[\log_a u]\,du = \dfrac{u'}{(\ln a)u}$ (p.354)

16 ...... $\displaystyle\int a^u\,du = \frac{a^u}{(\ln a)} + C$ (p.355)

17 ...... $\dfrac{d}{dx}[u^n] = nu^{n-1}u'$ (p. 355)

18 ...... $\displaystyle\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x = e$ (p. 356)

19 ...... If $\displaystyle\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\dfrac{\infty}{\infty}$, then by L'Hôpital's Rule
$\displaystyle\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$ (p. 363)

20 ...... Page 376:
$\dfrac{d}{dx}[\arcsin u] = \dfrac{u'}{\sqrt{1-u^2}}$
$\dfrac{d}{dx}[\arctan u] = \dfrac{u'}{1+u^2}$

21 ...... Page 382

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\frac{u}{a} + C$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\frac{u}{a} + C$$

22 ...... Summary of Basic Derivatives page 378

23 ...... Summary of Basic Integrals page 385

Delta Math Check List:

1 ...... 6 Practice Assignments and HW quizzes.

Khan Academy Check List:

1 ...... Derivatives of e^x and $\ln x$

2 ...... Derivatives of $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$

3 ...... Derivatives of Inverse functions

4 ...... Derivatives of Inverse Trig functions

5 ...... Indefinite integrals: e^x & $1/x$

6 ...... Integration Using Long Division

7 ...... Integration using completing the square

Always review your Notes and Examples (see topics if you lost your notes), Quizzes, and old homework problems. There is a separate pdf with Multiple choice practice as well.

1. Find the derivative
   (a) $f(x) = e^{2\ln(3x+1)}$

   (b) $f(x) = 5x^{-2} - [\ln\cos x - \ln(\sin x + x)]$

   (c) $f(x) = \dfrac{e^x + 9}{e^{x^2} - x^4}$

   (d) $f(x) = \ln(2x^2 + 1)$

   (e) $y = x^{\sqrt{2}}$

   (f) $y = x^x$

   (g) $f(x) = \dfrac{\sec x}{x}$

(h) $\ln y + xy^2 - 4x^3 + 10 = 3x$

(i) $f(x) = (x^2 + 6)\ln(3x)$

(j) $f(x) = \cot x$

(k) $y = x^{\tan x}$

(l) $y = \cos x(\tan x - \sec x)$

(m) $f(x) = 3^{4x}$

(n) $f(t) = \dfrac{3^{2t}}{t}$

(o) $y = \log_5 \dfrac{x^2 - 1}{x}$

(p) $g(t) = \log_2(t^2 + 7)^3$

2. Evaluate the integral.

   (a) $\int e^{\sec 2x} \sec 2x \tan 2x \; dx$

   (b) $\int \sec y(\tan y - \sec y) \; dy$

   (c) $\int e^{3x} \; dx$

   (d) $\int \tan^2 x + 1 \; dx$

(e) $\displaystyle\int \frac{(\ln x)^2}{x}\ dx$

(f) $\displaystyle\int \frac{x}{\sqrt{2x-1}}\ dx$

(g) $\displaystyle\int \frac{1}{3x+2}\ dx$

(h) $\displaystyle\int \cot x\ dx$

(i) $\displaystyle\int \frac{12}{1+9x^2}\ dx$

(j) $\displaystyle\int \frac{1}{\sqrt{-x^2-4x}}\ dx$
Hint: Complete the square

(k) $\displaystyle\int \frac{e^{2y}}{1-e^{2y}}\ dy$

(l) $\displaystyle\int \frac{e^{3x}-2e^{x}+5}{e^{2x}}\ dx$

(m) $\displaystyle\int 2^x\ dx$

(n) $\int_1^3 4^{x+1} + 2^x \; dx$

(o) $\int_1^3 \frac{e^{3/x}}{x^2} \; dx$

(p) $\int_0^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}} \; dx$

(q) $\int_{-2}^3 \frac{1}{x^2+4x+8} \; dx$
Hint: Complete the square

(r) $\int_0^{\pi/2} \frac{\cos x}{2^{\sin x}} \; dx$

3. Evaluate the limits, using L'Hôpital's Rule if necessary. If you do, remember to identify if it is $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form and state that you are using L'Hôpital's Rule.

   (a) $\displaystyle\lim_{x\to-3} \frac{3\sin(2x+6)}{3+x}$

   (b) $\displaystyle\lim_{x\to 3} \frac{3\ln(4-x)}{x-3}$

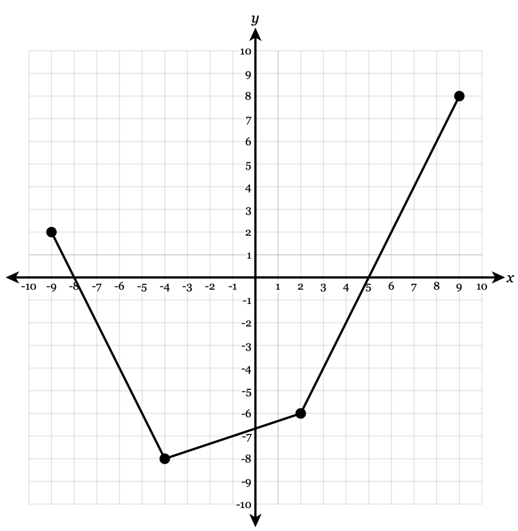
   (c) $\displaystyle\lim_{x\to\infty} \frac{\arctan x}{3}$

   (d) $\displaystyle\lim_{x\to 2} \frac{x^2-4}{x+2}$

   (e) $\displaystyle\lim_{x\to\infty} \frac{\ln x^2}{(\ln x)^2}$

   (f) $\displaystyle\lim_{x\to\infty} \frac{\ln 6x}{\ln 2x}$

4. The graph of the function f is shown below. Determine the value of $\displaystyle\lim_{x\to 2} \frac{f(2x)+2}{5x-10}$



5. Find an equation of the tangent line to $y = 5^{x-2}$ at the point $(2, 1)$

6. If $f(x) = \displaystyle\int_{\arctan x}^{2} 7^t \, dt$, then find $f'(x)$. (*Hint: FTC2 and the chain rule*)

7. (Calculator Active) The weight (in grams) of a bacterial culture at time t (hours) is modeled by the function

$$W(t) = \frac{1.25}{1 + 0.25e^{-0.4t}}$$

for time $t \geq 0$

(a) Find the weight after 1 hour.

(b) Find the rate at which the weight is increasing after 2 hours.

8. (Calculator Active) At what point (x, y) on the graph of $y = 2^x - 3$ does the tangent line have slope 21?

9. (No Calculator) A particle moves along the x axis so that at any time $t > 0$ its velocity is given by $v(t) = t \ln t - t$ At time $t = 1$, the position of the particle is $x(1) = 6$.
   (a) Write an expression for the acceleration of the particle.

   (b) For what values of t is the particle moving right?

   (c) What is the minimum velocity of the particle. Justify your conclusion.

   (d) If $\int t \ln t - t \; dt = \frac{1}{4}t^2(2 \ln t - 3) + C$, write an expression of the position $x(t)$ of the particle.

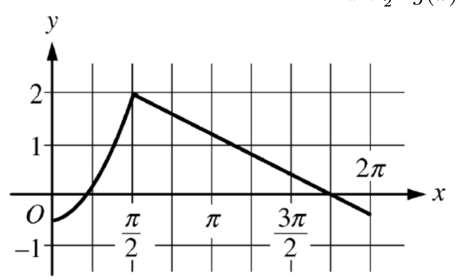
10. (No Calculator) Let $f(x) = e^x \cos x$.

(a) (1 point) Find the average rate of change of f on the interval $0 \leq x \leq \pi$.

(b) (2 points) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) (3 points) Find the absolute minimum value of f on the interval $0 \leq x \leq 2\pi$. Justify your answer.

(d) (3 points) Let g be a differentiable function such that $g(\frac{\pi}{2}) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{f(x)}{g(x)}$



Graph of g'